Oxford Cambridge and RSA

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A Level Mathematics B (MEI)

H640/03 Pure Mathematics and Comprehension

Sample Insert

Version 2

Date – Morning/Afternoon

Time allowed: 2 hours

Model Answers

INFORMATION FOR CANDIDATES

- This insert contains the article for Section B
- This document consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Archimedes's approximation of π

The constant π is defined to be the circumference of a circle divided by its diameter.

The value of π has been determined to an accuracy of more than twelve trillion decimal places. To the non-mathematician this may appear strange since it is not possible to measure the circumference and diameter of a circle to that degree of accuracy; this article explains how one of the greatest mathematicians of all time found the value of π to a high degree of accuracy without requiring any physical measurement.

Archimedes (287-212 BC) lived in Syracuse, Sicily. He developed many branches of mathematics, including calculus, in which he devised methods for finding areas under parabolas nearly 2000 years before Newton and Leibniz, and mechanics, in which he found the centres of gravity of various plane figures and solids and devised a method for calculating the weight of a body immersed in a liquid.

Whilst absorbed in a mathematical problem, Archimedes was killed by a soldier during the capture of Syracuse by the Romans.

Archimedes's method for determining the value of π is described below.

Fig. C1 shows a circle with unit radius and two regular hexagons.

The smaller regular hexagon has its vertices on the circle; it is called an *inscribed* polygon. Its perimeter is 6.

The larger regular hexagon has the midpoints of its edges on the circle; it is called an *escribed* polygon. Its perimeter is $4\sqrt{3}$.

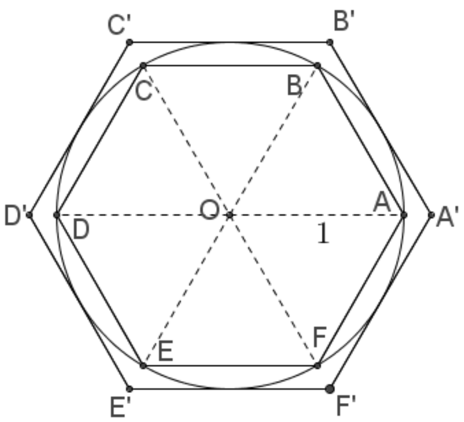

Fig. C1

The circumference of the circle is greater than the perimeter, ABCDEF, of the smaller hexagon but less than the perimeter, $A'B'C'D'E'F'$, of the larger hexagon.

Dividing the perimeters by the diameter of the circle gives lower and upper bounds for π of 3 and $2\sqrt{3}$, so that $3 < \pi < 2\sqrt{3}$.

To find tighter bounds, Archimedes repeatedly doubled the number of edges in the two regular polygons, from 6 to 12, 24, 48 and finally 96. The process of doubling the number of edges is described below.

Fig. C2 shows two adjacent vertices, P and Q, of a regular polygon inscribed in a circle with unit radius and centre O. PQ has length *a*. M is the midpoint of PQ. OM is extended to meet the circle at R. MR has length *h*. PR and RQ are adjacent edges of a regular polygon which has twice as many edges as the polygon which has PQ as an edge. PR has length *b*.

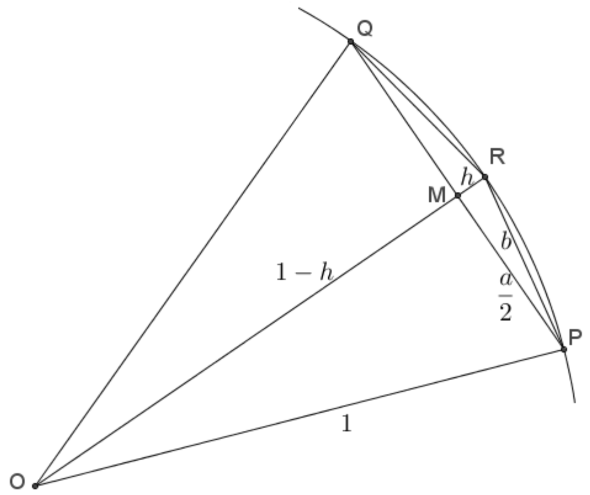

Fig. C2

Applying Pythagoras' Theorem

- to triangle OMP gives $1 = \dfrac{a^2}{4} + (1-h)^2$,
- to triangle PMR gives $b^2 = \dfrac{a^2}{4} + h^2$.

For the inscribed regular hexagon, $a = 1$. Substituting $a = 1$ in the equations above gives $h = \dfrac{2-\sqrt{3}}{2}$ and $b = \sqrt{2-\sqrt{3}}$. This can be written in the equivalent form $b = \dfrac{\sqrt{6}-\sqrt{2}}{2}$.

Therefore a regular polygon with 12 edges inscribed in a unit circle has edge length $\dfrac{\sqrt{6}-\sqrt{2}}{2}$.

Archimedes repeated this process to find the edge lengths of inscribed regular polygons with 24, 48 and 96 edges. He then used a similar technique for escribed regular polygons.

The inscribed and escribed regular polygons with 96 edges provide bounds for π which we now write, using decimal notation, as

$3.14103... < \pi < 3.14271...$.

Summary of Updates

Date	Version	Change
October 2018	2	We've reviewed the look and feel of our papers through text, tone, language, images and formatting. For more information please see our assessment principles in our "Exploring our question papers" brochures on our website.

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A Level Mathematics B (MEI)
H640/03 Pure Mathematics and Comprehension
Sample Question Paper Version 2.1

Date – Morning/Afternoon
Time allowed: 2 hours

You must have:
- Printed Answer Booklet
- the Insert

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **20** pages. The Question Paper consists of **12** pages.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r|<1$$

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1\, a^{n-1}b + {}^n\mathrm{C}_2\, a^{n-2}b^2 + \ \ldots\ + {}^n\mathrm{C}_r\, a^{n-r}b^r + \ \ldots\ + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ \ldots\ + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ \ldots \qquad (|x|<1,\ n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2\, x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient Rule $y = \frac{u}{v}$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h\to 0}\frac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$$

Integration by parts $\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small Angle Approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$

Numerical methods

Trapezium rule: $\int_a^b y\,dx \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving : $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B)$ **or** $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Sample Variance

$s^2 = \dfrac{1}{n-1} S_{xx}$ where $S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \dfrac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The Binomial Distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^nC_r p^r q^{n-r}$ where $q = 1 - p$

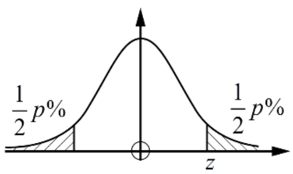
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \tfrac{1}{2}at^2$

$s = \tfrac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \tfrac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \tfrac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \tfrac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \tfrac{1}{2}\mathbf{a}t^2$

Answer **all** the questions

Section A (60 marks)

1 Express $\dfrac{2}{x-1}+\dfrac{5}{2x+1}$ as a single fraction. **[2]**

$$1 \quad \frac{2}{x-1} + \frac{5}{2x+1} = \frac{2(2x+1)+5(x-1)}{(x-1)(2x+1)}$$

$$= \frac{4x+2+5x-5}{(x-1)(2x+1)}$$

$$= \frac{9x-3}{(x-1)(2x+1)}$$

2 Find the first four terms of the binomial expansion of $(1-2x)^{\frac{1}{2}}$.

State the set of values of x for which the expansion is valid. **[4]**

$$2 \quad (1-2x)^{\frac{1}{2}} = 1 + \frac{(\frac{1}{2})(-2x)}{1} + \frac{(\frac{1}{2})(-\frac{1}{2})(-2x)^2}{2!} + \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})(-2x)^3}{3!}$$

$$= 1 - x - \frac{1}{2}x^2 - \frac{1}{2}x^3$$

Valid for $|-2x| < 1$

$|x| < \frac{1}{2}$

3 Show that points A (1, 4, 9), B (0, 11, 17) and C (3, –10, –7) are collinear. **[4]**

3 $\overrightarrow{AB} = B - A = \begin{pmatrix} 0-1 \\ 11-4 \\ 17-9 \end{pmatrix} = \begin{pmatrix} -1 \\ 7 \\ 8 \end{pmatrix}$

$\overrightarrow{AC} = C - A = \begin{pmatrix} 3-1 \\ -10-4 \\ -7-9 \end{pmatrix} = \begin{pmatrix} 2 \\ -14 \\ -16 \end{pmatrix} = -2\begin{pmatrix} -1 \\ 7 \\ 8 \end{pmatrix}$

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are parallel so A, B and C are collinear with common point A

4 Show that $\sum_{r=1}^{4} \ln \frac{r}{r+1} = -\ln 5$. **[3]**

4 $\sum_{r=1}^{4} \ln \frac{r}{r+1} = \ln \frac{1}{2} + \ln \frac{2}{3} + \ln \frac{3}{4} + \ln \frac{4}{5}$

$= \ln \left(\frac{1 \times \cancel{2} \times \cancel{3} \times \cancel{4}}{\cancel{2} \times \cancel{3} \times \cancel{4} \times 5} \right)$

$= \ln \left(\frac{1}{5} \right)$

$= -\ln 5$

5 **In this question you must show detailed reasoning.**

Fig. 5 shows the circle with equation $(x-4)^2+(y-1)^2=10$.
The points $(1, 0)$ and $(7, 0)$ lie on the circle. The point C is the centre of the circle.

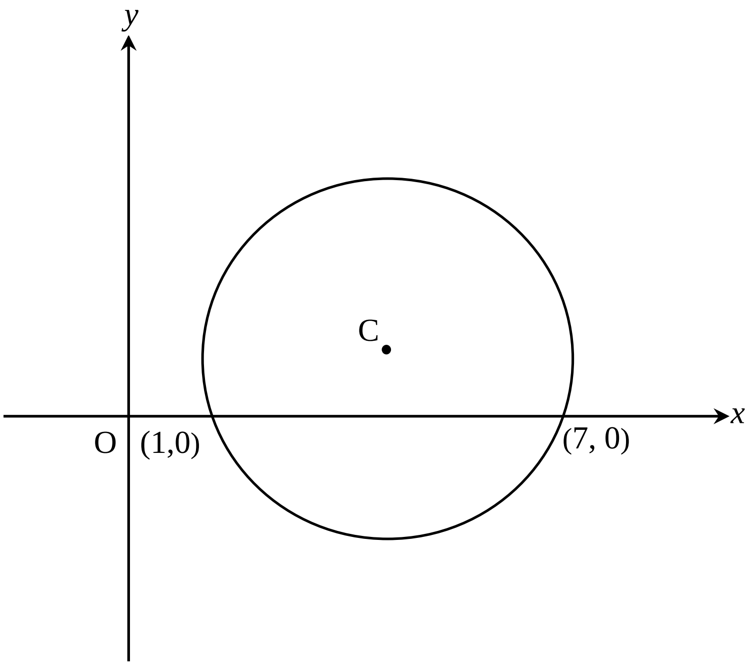
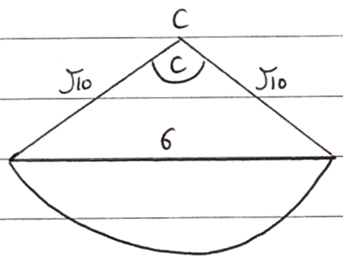


Fig. 5

Find the area of the part of the circle below the x-axis. **[5]**

5.



Cosine rule : $6^2 = (\sqrt{10})^2 + (\sqrt{10})^2 - 2(\sqrt{10})(\sqrt{10}) \cos C$

$\cos C = \dfrac{10 + 10 - 36}{2 \times 10}$

$\cos C = -0.8$

$C = 2.5^c$

Area of triangle $= \frac{1}{2} \times \sqrt{10} \times \sqrt{10} \times \sin 2.5$

$= 3$

Area under x axis = total area − area of triangle

$= \frac{1}{2} \times (\sqrt{10})^2 \times 2.5 - 3$

$= 12.49 - 3$

$= 9.49$

6 **Fig. 6** shows the curve with equation $y = x^4 - 6x^2 + 4x + 5$.

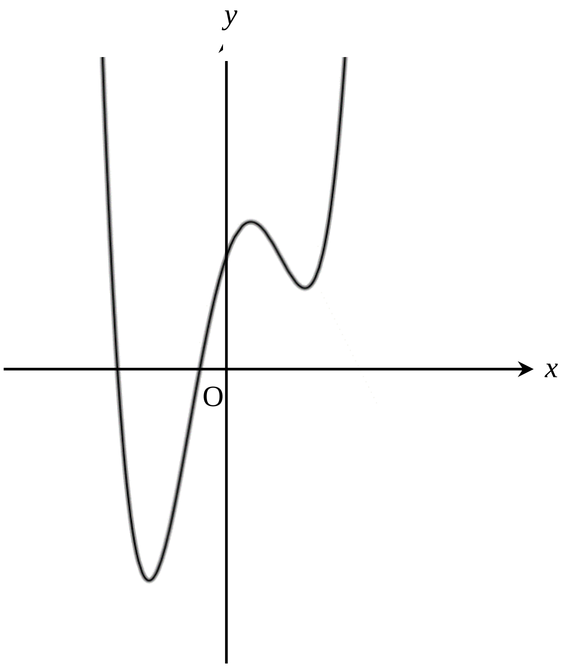


Fig. 6

Find the coordinates of the points of inflection. **[5]**

6. $y = x^4 - 6x^2 + 4x + 5$

$\frac{dy}{dx} = 4x^3 - 12x + 4$

$\frac{d^2y}{dx^2} = 12x^2 - 12$

A point of inflection is where $\frac{d^2y}{dx^2} = 0$

$0 = 12x^2 - 12$

$x^2 = 1$

$x = \pm 1$

At $x = 1$, $y = 1 - 6 + 4 + 5 = 4$

At $x = -1$, $y = 1 - 6 - 4 + 5 = -4$

$\therefore$ Coordinates are $(1, 4)$ and $(-1, -4)$

7 By finding a counter example, disprove the following statement.

If p and q are non-zero real numbers with $p < q$, then $\frac{1}{p} > \frac{1}{q}$. **[2]**

7 eg. $p = -1$, $q = 1$

$\frac{1}{p} = -1$ $\frac{1}{q} = 1$

$\frac{1}{p} < \frac{1}{q}$ So this is a counter example

8 In **Fig. 8**, OAB is a thin bent rod, with OA = 1 m, AB = 2 m and angle OAB = 120°.
Angles , ϕ and h are as shown in **Fig. 8**.

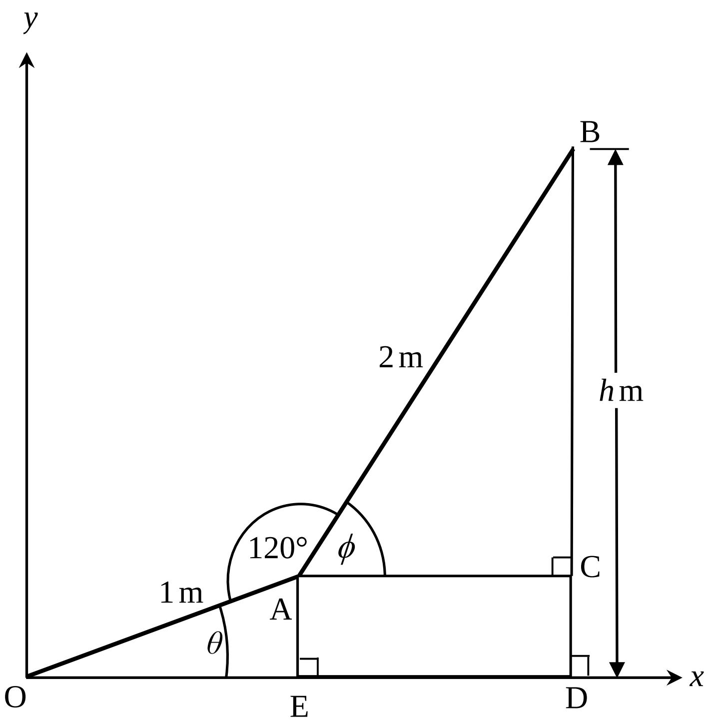


Fig. 8

(a) Show that $h = \sin\theta + 2\sin(\theta + 60°)$. **[3]**

The rod is free to rotate about the origin so that and ϕ vary. You may assume that the result for h in part **(a)** holds for all values of .

8 a) $\angle OAE = 90 - \theta$

$\phi = 360 - 120 - \angle OAE - 90$

$\phi = 150 - (90 - \theta)$

$\phi = 60 + \theta$

$\Rightarrow \frac{BC}{2} = \sin\phi$

$BC = 2\sin(60 + \theta)$

$\sin\theta = \frac{AE}{1}$

$AE = \sin\theta$

$AE = CD = \sin\theta$

$\therefore \quad h = CD + BC$

$h = \sin\theta + 2\sin(\theta + 60)$

(b) Find an angle θ for which $h = 0$. **[5]**

b) $h = \sin\theta + 2\sin(\theta + 60)$

$0 = \sin\theta + 2\sin\theta\cos 60 + 2\sin 60\cos\theta$

$0 = \sin\theta + \sin\theta + \sqrt{3}\cos\theta$

$0 = 2\sin\theta + \sqrt{3}\cos\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{-\sqrt{3}}{2}$

$\tan\theta = \frac{-\sqrt{3}}{2}$

$\theta = -40.9$

This is $40.9°$ degrees below the horizontal

9 **(a)** Express $\cos\theta + 2\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $0 < \alpha < \frac{1}{2}\pi$ and R is positive and given in exact form. **[4]**

The function is defined by $f(\theta) = \dfrac{1}{(k + \cos\theta + 2\sin\theta)}$, $0 \le \theta \le 2\pi$, k is a constant.

9 a) $\cos\theta + 2\sin\theta = R\cos(\theta - \alpha)$

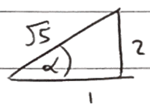
$\cos\theta + 2\sin\theta = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$1 = R\cos\alpha \qquad 2 = R\sin\alpha$

$\dfrac{R\sin\alpha}{R\cos\alpha} = \dfrac{2}{1}$

$\tan\alpha = 2$

$\alpha = 1.107^c$



$R = \dfrac{1}{\cos\alpha} = \dfrac{1}{\frac{1}{\sqrt{5}}} = \sqrt{5}$

$\therefore \cos\theta + 2\sin\theta = \sqrt{5}\cos(\theta - 1.107)$

(b) The maximum value of $f(\theta)$ is $\dfrac{(3+\sqrt{5})}{4}$.

Find the value of k. **[3]**

b) $f(\theta) = \dfrac{1}{(k + \cos\theta + 2\sin\theta)} = \dfrac{1}{k + \sqrt{5}\cos(\theta - 1.107)}$

The max of $f(\theta)$ is when $\cos(\theta - 1.107) = -1$

$$\max = \frac{1}{k - \sqrt{5}}$$

$$\frac{3 + \sqrt{5}}{4} = \frac{1}{k - \sqrt{5}} \times \frac{k + \sqrt{5}}{k + \sqrt{5}}$$

$$\frac{3 + \sqrt{5}}{4} = \frac{k + \sqrt{5}}{k^2 - 5}$$

$$k = 3$$

10 The function $f(x)$ is defined by $f(x) = x^4 + x^3 - 2x^2 - 4x - 2$.

(a) Show that $x = -1$ is a root of $f(x) = 0$. **[1]**

10 a) $f(x) = x^4 + x^3 - 2x^2 - 4x - 2$

$f(-1) = (-1)^4 + (-1)^3 - 2(-1)^2 - 4(-1) - 2$

$= 1 - 1 - 2 + 4 - 2$

$= 0$

hence $x = -1$ is a root

(b) Show that another root of $f(x) = 0$ lies between $x = 1$ and $x = 2$. **[2]**

b) $f(1) = 1 + 1 - 2 - 4 - 2 = -6$

$f(2) = 16 + 8 - 8 - 8 - 2 = 6$

change of sign indicates root

(c) Show that $f(x)=(x+1)g(x)$, where $g(x)=x^3+ax+b$ and a and b are integers to be determined. **[3]**

c)

$$\begin{array}{r|l} & x^3 - 2x - 2 \\ x+1 & x^4 + x^3 - 2x^2 - 4x - 2 \\ & x^4 + x^3 \\ & -2x^2 - 4x - 2 \\ & -2x^2 - 2x \\ & -2x - 2 \\ & -2x - 2 \\ & 0 \end{array}$$

$\therefore \quad f(x) = (x+1)(x^3 - 2x - 2)$

(d) Without further calculation, explain why $g(x)=0$ has a root between $x=1$ and $x=2$. **[1]**

$f(x) = (x+1)g(x)$

The LHS = 0 for the root between 1 and 2

So the RHS must also equal zero

Hence $g(x) = 0$ for this root

(e) Use the Newton-Raphson formula to show that an iteration formula for finding roots of $g(x)=0$ may be written

$$x_{n+1}=\frac{2x_n^{\,3}+2}{3x_n^{\,2}-2}.$$

$x=1$

Determine the root of $g(x)=0$ which lies between $x=1$ and $x=2$ correct to 4 significant figures. **[3]**

$x_{n+1} = x_n - \dfrac{g(x_n)}{g'(x_n)}$

$= x_n - \dfrac{x_n^3 - 2x_n - 2}{3x_n^2 - 2}$

$= \dfrac{3x_n^3 - 2x_n - x_n^3 + 2x_n + 2}{3x_n^2 - 2}$

$= \dfrac{2x_n^3 + 2}{3x_n^2 - 2}$

$x_0 = 1.5$

$x_1 = 1.842$

$x_2 = 1.773$

$x_3 = 1.769$

$x_4 = 1.769$

hence the root = 1.769 to 4sf

11 The curve $y = \mathrm{f}(x)$ is defined by the function $\mathrm{f}(x) = \mathrm{e}^{-x}\sin x$ with domain $0 \le x \le 4\pi$.

(a) **(i)** Show that the x-coordinates of the stationary points of the curve $y = \mathrm{f}(x)$, when arranged in increasing order, form an arithmetic sequence.

11 a) i. $f(x) = e^{-x}\sin x$

$f'(x) = e^{-x}\cos x - e^{-x}\sin x$

Stationary point is when $f'(x) = 0$

$0 = e^{-x}\cos x - e^{-x}\sin x$

$0 = e^{x}(\cos x - \sin x)$

$e^{x} \neq 0$ so $\cos x - \sin x = 0$

$\cos x = \sin x$

$\tan x = 1$

$x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$

This is an arithmetic sequence with $a = \frac{\pi}{4}$ and $d = \pi$

(ii) Show that the corresponding y-coordinates form a geometric sequence. **[9]**

when $x = \frac{\pi}{4}$, $y = e^{-\frac{\pi}{4}} \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} e^{-\frac{\pi}{4}}$

$x = \frac{5\pi}{4}$, $y = e^{-\frac{5\pi}{4}} \sin\frac{5\pi}{4} = -\frac{\sqrt{2}}{2} e^{-\frac{5\pi}{4}}$

$x = \frac{9\pi}{4}$, $y = e^{-\frac{9\pi}{4}} \sin\frac{9\pi}{4} = \frac{\sqrt{2}}{2} e^{-\frac{9\pi}{4}}$

This is a geometric sequence with $a = \frac{\sqrt{2}}{2} e^{-\frac{\pi}{4}}$ and $r = -e^{-\pi}$

(ii) Show that the corresponding *y*-coordinates form a geometric sequence. **[9]**

when $x = \frac{\pi}{4}$, $y = e^{-\frac{\pi}{4}} \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} e^{-\frac{\pi}{4}}$

$x = \frac{5\pi}{4}$, $y = e^{-\frac{5\pi}{4}} \sin\frac{5\pi}{4} = -\frac{\sqrt{2}}{2} e^{-\frac{5\pi}{4}}$

$x = \frac{9\pi}{4}$, $y = e^{-\frac{9\pi}{4}} \sin\frac{9\pi}{4} = \frac{\sqrt{2}}{2} e^{-\frac{9\pi}{4}}$

This is a geometric sequence with $a = \frac{\sqrt{2}}{2} e^{-\frac{\pi}{4}}$

and ~~k=~~ $r = -e^{-\pi}$

(b) Would the result still hold with a larger domain? Give reasons for your answer. **[1]**

b) Yes, the sequences would have more elements but would have the same common ratio and difference

Answer **all** the

questions **Section B** (15

marks)

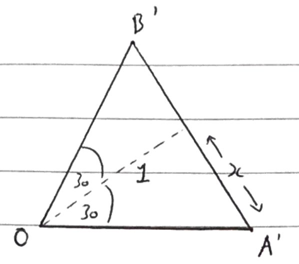
The questions in this section refer to the article on the Insert. You should read the article before attempting the questions.

12 Explain why the smaller regular hexagon in **Fig. C1** has perimeter 6. **[1]**

12 Each triangle is equilateral with sides of length 1 (like OAB). There are 6 of these so the total perimeter is 6

13 Show that the larger regular hexagon in **Fig. C1** has perimeter $4\sqrt{3}$. **[3]**

13



$\tan 30 = \frac{x}{1}$

$\frac{1}{\sqrt{3}} = x$

total perimeter $= 6 \times 2x$

$= \frac{12}{\sqrt{3}}$

$= 4\sqrt{3}$

14 Show that the two values of b given on line 36 are equivalent. **[3]**

14. $\left(\dfrac{\sqrt{6}-\sqrt{2}}{2}\right)^2 = \dfrac{6-2\sqrt{12}+2}{4}$

$= \dfrac{8-2\sqrt{12}}{4}$

$= \dfrac{8-4\sqrt{3}}{4}$

$= 2-\sqrt{3}$

$\left(\sqrt{2-\sqrt{3}}\right)^2 = 2-\sqrt{3}$

hence $\dfrac{\sqrt{6}-\sqrt{2}}{2} = \sqrt{2-\sqrt{3}}$ since they are positive

15 **Fig. 15** shows a unit circle and the escribed regular polygon with 12 edges.

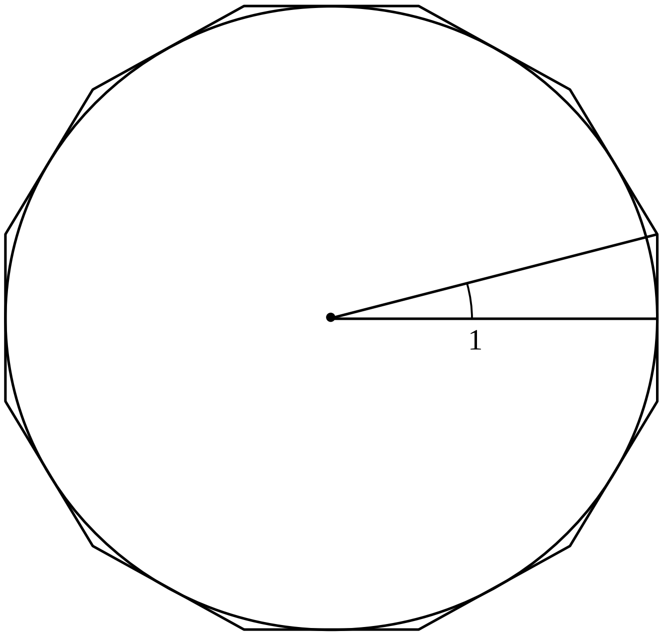
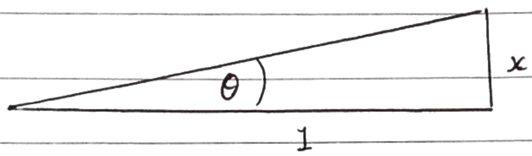


Fig. 15

(a) Show that the perimeter of the polygon is $24\tan 15°$. **[2]**

15. a)



$$\theta = \frac{360}{24} = 15$$

$$\tan 15 = \frac{x}{1}$$

$$x = \tan 15$$

$$\text{Perimeter} = 12 \times 2x$$
$$= 24 \tan 15$$

(b) Using the formula for $\tan(\theta - \phi)$ show that the perimeter of the polygon is $48 - 24\sqrt{3}$. **[3]**

b) Perimeter $= 24 \tan 15$

$= 24 \tan (45 - 30)$

$= 24 \left(\frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30} \right)$

$= 24 \left(\frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} \right)$

$= 24 \left(\frac{\sqrt{3} - 1}{\sqrt{3} + 1} \right) \times \frac{-\sqrt{3} + 1}{-\sqrt{3} + 1}$

$= 24 \left(\frac{-3 + 2\sqrt{3} - 1}{-3 + 1} \right)$

$= 24 \left(\frac{2\sqrt{3} - 4}{-2} \right)$

$= 48 - 24\sqrt{3}$

16 On a unit circle, the inscribed regular polygon with 12 edges gives a lower bound for π, and the escribed regular polygon with 12 edges gives an upper bound for π.

Calculate the values of these bounds for π, giving your answers:

(i) in surd form

[3]

16 i) Lower bound : $3(\sqrt{6} - \sqrt{2})$
Upper bound : $24 - 12\sqrt{3}$

(ii) correct to 2 decimal places.

ii) Lower bound = 3.11
Upper bound = 3.22

END OF QUESTION PAPER